The Value of Control and the Costs of Illiquidity

Rui Albuquerque and Enrique Schroth
The model

• One incumbent blockholder $I$ holds a proportion of $\alpha < 1$ of the firm, and the remaining $1 - \alpha$ are held by dispersed shareholders.
• The firm’s cash flow is an r.v. that takes value on a grid $\{\pi_1, ..., \pi_N\}$. It evolves stochastically according to $\Pr[\pi' = \pi_m | \pi = \pi_l] = q_{lm}, q_{lm} > 0$ and $\sum_{m=1}^{N} q_{lm} = 1$.
• Denote by $v(\pi^I_l)$ the incumbent’s per share value of the block at $\pi^I_l$. The block holder also obtains per share private benefits $B$. 
The model

- At the beginning of every period, \( I \) faces a liquidity shock with probability \( \theta \). If the shock occurs, then \( I \) is forced to sell to a rival blockholder \( R \) at a price of \( \phi v(\pi_{m}^{R}) \).
- Therefore, the ex ante block price upon a liquidity shock is

\[
L_{I}^{v} = \phi \sum_{k=1}^{N} q_{lk} v(\pi_{k})
\]

- If liquidity shock doesn’t occur, the incumbent is matched with a potential buyer with probability \( \eta \). Assume trading is result of Nash bargaining, and the seller’s bargaining power is \( \psi \). If bargaining is successful, \( R \) pays \( s(\pi_{k}^{I}, \pi_{m}^{R}) \) and gets \( v(\pi_{m}^{R}) \) plus the private benefit.
The model

- The value of the block to the incumbent is

\[ v(\pi^I_l) = \pi^I_l + \delta \left[ (1 - \theta) \sum_{k=1}^{N} q_{lk} \tilde{v}_l(\pi^I_k) + \theta L^v_l \right] \]

Where \( \tilde{v}_l(\pi^I_k) + B = \eta \sum_{m=1}^{N} q_{lm} \max \left[ s(\pi^I_k, \pi^R_m), v(\pi^I_k) + B \right] + (1 - \eta) [v(\pi^I_k) + B] \)
The model

- The model assumes complete information by all investors
- Therefore, dispersed shareholders trade at

\[ p(\pi^I_l) = \pi^I_l + \delta \left[ (1 - \theta) \sum_{k=1}^{N} q_{lk} \tilde{p}_l(\pi^I_k) + \theta L^P_l \right] \]

where \( \tilde{p}_l(\pi^I_k) = p(\pi^I_k) + \eta \sum_{m=1}^{N} q_{lm} \max[p(\pi^R_m) - p(\pi^I_k), 0] \)

and \( L^P_l = \sum_{k=1}^{N} q_{lk} p(\pi_k) \)
The model

• The block premium and cumulative abnormal returns are as follows

\[ BP(\pi_k^I, \pi_m^R) \equiv \left\{ \begin{array}{ll}
\frac{\phi_U(\pi_m^R)}{p(\pi_k^I)} - 1, & \text{if a liquidity shock occurs}, \\
\frac{s(\pi_k^I, \pi_m^R)}{p(\pi_k^I)} - 1, & \text{otherwise}.
\end{array} \right. \]

\[ CAR(\pi_k^I, \pi_m^R) \equiv \frac{p(\pi_m^R)}{p(\pi_k^I)} - 1 \]
The model

- After adding some further restrictions and solving the model, we have

\[ B_i = b_0 + b_1 E(\nu(\pi_i)) + b_2 E(p(\pi_i))(1 - \alpha_i)/\alpha_i \]

\[ BP(\pi^R_k, \pi^R_m) \equiv \begin{cases} \frac{\phi p(\pi^R_m)}{p(\pi^R_k)} - 1, & \text{if a liquidity shock occurs,} \\ \frac{s(\pi^L_k, \pi^R_m)}{p(\pi^L_k)} - 1, & \text{otherwise.} \end{cases} \]

\[ \mathbf{v} = \pi^T \left\{ I - \delta \left[ (1 - \theta)(Q^T + \eta \psi M^2) + \theta \phi Q^T \right] \right\}^{-1} \]

\[ p = \pi^T \left\{ I - \delta \left[ Q^T + (1 - \theta) \eta M^2 \right] \right\}^{-1} \]

\[ CAR(\pi^L_k, \pi^R_m) \equiv \frac{p(\pi^R_m)}{p(\pi^L_k)} - 1 \]

\[ \theta(x_i, \beta) = \frac{\exp(x_i' \beta)}{1 + \exp(x_i' \beta)} \]

\[ \phi(z_i, \gamma) = \frac{\exp(z_i' \gamma)}{1 + \exp(z_i' \gamma)} \]
Estimating the model

• **Q**: estimate an AR(1) process of the average yearly cash flow, and discretize it applying Tauchen and Hussey (1991)

• Use MSM to estimate $\Gamma = \{\psi, \eta, b_0, b_1, b_2, \beta, \gamma\}$. $\hat{\Gamma}$ minimizes

$$J(\Gamma) = (m(. , \Gamma) - M)'W(m(. , \Gamma) - M)$$

Where $m(\{BP_i, CAR_i, x_i, z_i\}; \Gamma)$ is a vector of moments
Estimating the model

• With initial condition $\Gamma(0)$, calculate $\theta$ and $\phi$
• Solve for $v(\pi)$ and $p(\pi)$
• Compute models’ CAR for all possible states and choose the grid values such that the distance between actual CAR and $\frac{p(\pi_m)}{p(\pi_i)} - 1$ is minimized
• Compute the block premium
• Compute the moment conditions and evaluate $J(\Gamma)$
• 114 acquisitions of blocks of more than 35% but less than 90%
• Average BP is 6.8% and average CAR is 9.6%
• 47% negative BP and 42% negative CAR
• BP and CAR correlation: 0.37
Identification: $\theta$

All trades with $\text{CAR} < 0$ must have been caused by a liquidity shock.

Any trade with $\text{BP} \geq \text{CAR} > 0$ cannot be due to liquidity shock.

For $\text{CAR} > 0$ and $\text{CAR} > \text{BP}$, the model assigns an ex-post probability that a liquidity shock occurred.

\[
\frac{\theta \times \Pr[\pi' > \pi_l | \pi = \pi_l]}{\theta \times \Pr[\pi' > \pi_l | \pi = \pi_l] + (1 - \theta) \eta}
\]
Identification: $\theta$

The spread between BP and CAR produces information about $\theta$ even for trades not caused by a liquidity shock.
Identification: $\phi$

- Fire sale prices affect only the blockholders’ value.
- Therefore, when a liquidity shock occurs, the model assigns the “extra” variation in block prices to variation in $\phi$.

Similarly, the spread between BP and CAR produces information about $\phi$ even for trades not caused by a liquidity shock.
Identification: $\eta$

Spread between BP and CAR is not informative of $\eta$

Identification relies on the fact that it cannot explain the occurrence of negative price reactions
Moment conditions

Conditional on liquidity shock
• $E(BP|CAR < 0, BP < 0)$
• $VAR(BP|CAR < 0, BP < 0)$
• $VAR(CAR|CAR < 0, BP < 0)$
• $E(BP \times CAR|CAR < 0, BP < 0)$
• $E(BP \times z|CAR < 0, BP < 0)$

Conditional on no liquidity shock
• $E(BP - CAR|BP \geq CAR > 0)$
• $VAR(BP - CAR|BP \geq CAR > 0)$
• $VAR(CAR|BP \geq CAR > 0)$
• $VAR(BP|BP \geq CAR > 0)$

Unconditional
• $E(BP \times x)$
• $E(CAR \times x)$
• $E(BP \times z)$
• $E(BP \times z)$
Panel A: Model fit

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th></th>
<th>CAR</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Predicted</td>
<td>Actual</td>
<td>Predicted</td>
</tr>
<tr>
<td>Mean</td>
<td>0.067</td>
<td>0.101</td>
<td>0.096</td>
<td>0.022</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.584</td>
<td>0.468</td>
<td>0.319</td>
<td>0.078</td>
</tr>
<tr>
<td>Median</td>
<td>0.035</td>
<td>0.058</td>
<td>0.050</td>
<td>0.016</td>
</tr>
<tr>
<td>Proportion of negatives</td>
<td>0.465</td>
<td>0.412</td>
<td>0.421</td>
<td>0.421</td>
</tr>
<tr>
<td>corr(Actual, Predicted)</td>
<td>0.121</td>
<td>0.393</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th></th>
<th>(2)</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Over-identifying restrictions test(^b)</td>
<td>(\chi^2)</td>
<td>(p) value</td>
<td>(\chi^2)</td>
<td>(p) value</td>
</tr>
<tr>
<td></td>
<td>28.28</td>
<td>0.34</td>
<td>1,557.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| Joint significance test\(^c\) | \(\chi^2\) | \(p\) value | \(\chi^2\) | \(p\) value |
| J                             | 41.93 | 0.07    | 1,953.21 | 0.00   |

- The model estimates BP better than CAR
- Reject that all coefficients are 0
- Can not reject that the model is over-identified at 95%
## Illiquidity discounts

<table>
<thead>
<tr>
<th></th>
<th>Sample mean</th>
<th>Standard deviation</th>
<th>Min</th>
<th>First quartile</th>
<th>Median</th>
<th>Third quartile</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owner’s liquidity parameter ($\theta$)</td>
<td>0.198</td>
<td>0.297</td>
<td>0.000</td>
<td>0.008</td>
<td>0.045</td>
<td>0.244</td>
<td>0.999</td>
</tr>
<tr>
<td>Asset’s liquidity parameter ($\phi$)</td>
<td>0.921</td>
<td>0.097</td>
<td>0.587</td>
<td>0.889</td>
<td>0.966</td>
<td>0.995</td>
<td>1.000</td>
</tr>
<tr>
<td>Marketability discount</td>
<td>0.131</td>
<td>0.222</td>
<td>0.002</td>
<td>0.010</td>
<td>0.024</td>
<td>0.125</td>
<td>0.887</td>
</tr>
<tr>
<td>($1 - \frac{\psi(\theta, \phi, \eta)}{\psi(\theta=0, \phi, \eta=1,.)}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Illiquidity spillover discount</td>
<td>0.021</td>
<td>0.015</td>
<td>0.003</td>
<td>0.012</td>
<td>0.017</td>
<td>0.027</td>
<td>0.097</td>
</tr>
<tr>
<td>($1 - \frac{\rho(\theta, \phi, \eta,.)}{\rho(\theta=0, \phi, \eta=1,.)}$)</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Control discount</td>
<td>0.125</td>
<td>0.223</td>
<td>0.001</td>
<td>0.005</td>
<td>0.016</td>
<td>0.110</td>
<td>0.886</td>
</tr>
<tr>
<td>($1 - \frac{\phi(\theta, \phi, \eta,.)}{\phi(\theta, \phi, \eta,.)}$)</td>
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</tbody>
</table>

Discount factor $\delta = 1/1.1$
### Illiquidity Discounts by Industry

<table>
<thead>
<tr>
<th>Major Group</th>
<th>Top 5</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>Air Transportation</td>
<td>3</td>
<td>0.462</td>
<td>0.432</td>
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<tr>
<td>15</td>
<td>Building Contractors</td>
<td>4</td>
<td>0.233</td>
<td>0.323</td>
</tr>
<tr>
<td>13</td>
<td>Oil And Gas Extraction</td>
<td>8</td>
<td>0.219</td>
<td>0.265</td>
</tr>
<tr>
<td>20</td>
<td>Food And Kindred Products</td>
<td>4</td>
<td>0.156</td>
<td>0.255</td>
</tr>
<tr>
<td>80</td>
<td>Health Services</td>
<td>5</td>
<td>0.155</td>
<td>0.252</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Major Group</th>
<th>Bottom 5</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>Insurance Carriers</td>
<td>3</td>
<td>0.024</td>
<td>0.009</td>
</tr>
<tr>
<td>73</td>
<td>Business Services</td>
<td>5</td>
<td>0.022</td>
<td>0.013</td>
</tr>
<tr>
<td>36</td>
<td>Electronic And Other Equipment</td>
<td>3</td>
<td>0.017</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>Electrical Equipment (Except Computers)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>Electric, Gas, And Sanitary Services</td>
<td>4</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>60</td>
<td>Depository Institutions</td>
<td>3</td>
<td>0.009</td>
<td>0.000</td>
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</tbody>
</table>
Conclusion

• The estimates of the average marketability discount is large, but also very variant
• Illiquidity spillover on disperse shareholders is considerable
• This paper also provides a method to measure control discount